FIRST PUBLIC EXAMINATION

Preliminary Examination in Philosophy, Politics and Economics

INTRODUCTION TO PHILOSOPHY

TRINITY TERM 2019

Thursday 27 June, 9.30 am - 12.30 pm

This paper contains three sections: Logic; General Philosophy; and Moral Philosophy. You must answer FOUR questions, including at least one question from each section. You may answer your fourth question from any section.

In the Logic section, questions 1 and 2 are of an ‘elementary and straightforward’ nature; the remaining questions are more demanding. You may answer only one of questions 1 and 2 (but are not obliged to attempt either). The numbers in the margin in the Logic section indicate the marks which the examiners expect to assign to each part of the question.

IMPORTANT

You must use a separate booklet for each answer.

Write your CANDIDATE NUMBER on each booklet. DO NOT write your name.

Do NOT turn over until told that you may do so.
1. (a) Formalize each of the following English sentences first in $\mathcal{L}_1$ and then in $\mathcal{L}_2$. Your formalizations should be as detailed as possible. Comment on any difficulties and points of interest.

   (i) Every dog barks and has a tail. [4]
   (ii) Every dog is male or female. [4]
   (iii) Each of Jack, Jill, and Henry owns a bike. [4]

(b) Explain what it is for a connective in English to be truth-functional. Use examples to illustrate your explanation. [4]

(c) If $R$ is a binary relation, its converse, $R^C$, is defined as follows:

   $$R^C = \{ \langle d, e \rangle : \langle e, d \rangle \in R \}.$$

Determine whether the following are true or false, in each case justifying your answer.

   (i) If $R$ is transitive, so is $R^C$. [3]
   (ii) If $R$ is nonempty and $R = R^C$, then there is at least one nonempty set on which $R$ is reflexive. [3]
   (iii) $R^{C^C}$, i.e. the converse of the converse of $R$, is $R$ itself. [3]
2. (a) List as many ways as possible in which quotation marks may be added to the following expressions to obtain true English declarative sentences. Explain your answers, commenting on any difficulties or points of interest.

(i) The sun rises in the morning is a true sentence. [1]
(ii) Cambridge denotes Cambridge, and Cambridge denotes a word, although Cambridge is a city. [2]
(iii) The expression ‘Newcastle’ contains both an opening and a closing quotation mark. [2]
(iv) Contains more than five words starts with a vowel and contains more than five words. [2]
(v) Can something that is a sentence fragment and so is not a question like this be a question. [3]

(b) (i) What is it for a set of $\mathcal{L}_1$-sentences to be semantically consistent? [1]
(ii) What is it for an $\mathcal{L}_1$-argument to be valid? [2]
(iii) Where $\phi_1, \phi_2$, and $\psi$ are $\mathcal{L}_1$-sentences, can it ever be the case that $\phi_1 \models \psi$ but not $\phi_1, \phi_2 \models \psi$? Explain why or why not. [2]

(c) Formalize the following passage as a valid argument in $\mathcal{L}_1$, rewording the premisses as needed and adding any additional premisses on which the speaker is implicitly relying. Demonstrate the validity of the argument using a truth table or natural deduction. Specify your dictionary carefully and note any difficulties or points of interest.

Smith always carries an umbrella or wears a sunhat, but never both. And if he’s not wearing a coat, then he’s not carrying an umbrella. If it’s winter and Smith’s wearing a sunhat, he’s a hypochondriac. So Smith must be wearing a coat, since he’s no hypochondriac and it’s December. [10]
3. (a) Given an $\mathcal{L}_2$-structure $\mathcal{A}$, what is a variable assignment over $\mathcal{A}$? [1]

(b) Let $\mathcal{A}$ be some $\mathcal{L}_2$-structure. Decide whether the following claims are true. Justify your answers.

(i) If $|P|_{\mathcal{A}} = T$, then there is a constant $\kappa$ such that $|P\kappa|_{\mathcal{A}} = T$. [2]

(ii) If $|P|_{\mathcal{A}} = \mathcal{A}$, then for all variable assignments $\alpha$ over $\mathcal{A}$ and all variables $v$, $|Pv|_{\mathcal{A}} = T$. [2]

(iii) If $|P|_{\mathcal{A}}$ is nonempty, then for all variable assignments $\alpha$ over $\mathcal{A}$, there is a variable $v$ such that $|Pv|_{\mathcal{A}} = T$. [2]

(c) Consider an $\mathcal{L}_2$-structure $\mathcal{A}$ satisfying the following conditions

- $\mathcal{D}_\mathcal{A} = \{1, 2, 3, 4\}$;
- $|R|_\mathcal{A} = \{\langle d, e \rangle : \text{there is an arrow going directly from } d \text{ to } e \text{ in the diagram below}\}$.

For each of the following formulae, specify a variable assignment over $\mathcal{A}$ on which the formula is true. Briefly explain why the formula is true on the assignment you have specified.

(i) $Rxx$ [2]

(ii) $\neg \exists y Rxy$ [2]

(iii) $\exists y (\neg Rxy \land \neg Ryx)$ [3]

(iv) $\forall y (\neg Rxy \rightarrow \neg Ryx)$ [3]

(v) $\forall y (\exists z Ryz \leftrightarrow Ryx)$ [3]

(d) The expression $\phi(x)$ will be used to mean an $\mathcal{L}_2$-formula in which $x$ but no other variable occurs freely. Furthermore, if $\mathcal{S}$ is an $\mathcal{L}_2$-structure and $d$ an element in its domain, then $\phi(x)$ is said to define $d$ relative to $\mathcal{S}$ if and only if: $|\phi(x)|_{\mathcal{S}} = T$ if and only if $\alpha(x) = d$.

(i) Returning to structure $\mathcal{A}$ from part (c), does the formula $\forall y Rxy$ define anything relative to $\mathcal{A}$? Justify your answer. [2]

(ii) Is there an $\mathcal{L}_2$-formula which defines 3 relative to structure $\mathcal{A}$ from part (c)? Justify your answer. [3]
4. (a) Are the following strings correct abbreviation of $\mathcal{L}_2$-formulae? Are they $\mathcal{L}_2$-sentences? If an expression is an $\mathcal{L}_2$-formula,

- supply any indices and brackets that have been dropped in accordance with abbreviation conventions;
- indicate which occurrences of variables are bound and which are free; and
- indicate the scope of each quantifier.

(i) $\forall x \forall a (Rxx \rightarrow Rxa)$  
(ii) $\forall x \forall y \forall y \forall x (R^3xyz \rightarrow Pz)$  
(iii) $\forall x_{17} Q_3 \land P_2$  
(iv) $\forall x R^2x \rightarrow P_7x \lor Qy$  
(v) $\forall x \exists x (Pxx \lor Pxxx \lor Pxxxx)$

(b) Consider the following variation on the rule of existential elimination:

$(\exists\text{Elim}^*):$ Assume that $\phi$ is a formula with at most $v$ occurring freely in which the constant $t$ does not occur. If there is a proof of $\psi$ with no undischarged assumption other than $\phi[t/v]$ and a proof of $\exists v \phi$, then the result of appending $\psi$ to these proofs and discharging all assumptions of $\phi[t/v]$ is a proof of $\psi$.

In other words,

\[ [\phi[t/v]] \]
\[ \vdots \]
\[ \exists v \phi \quad \psi \]

provided that $t$ does not occur in $\exists v \phi$ or in any undischarged assumption other than $\phi[t/v]$ in the proof of $\psi$.

(i) Explain the difference between this rule and $(\exists\text{Elim})$.  
(ii) Would $\mathcal{L}_2$ still be sound if $(\exists\text{Elim}^*)$ were used in place of $(\exists\text{Elim})$? If so, explain why. If not, provide an example of an invalid inference that could be proved using $(\exists\text{Elim}^*)$.

(c) Determine whether each of the following $\mathcal{L}_2$-inferences is valid. If it is valid, provide a proof in natural deduction (which suffices to establish validity by the adequacy theorem). If it is not valid, provide a counterexample. You do not need to prove that your counterexample is a counterexample.

(i) $\forall x Pxa \lor \exists x Pxa \vdash Pba$  
(ii) $\vdash \forall x \exists y Rx \rightarrow \neg \forall x \neg Rxx$  
(iii) $\forall x \exists y (Rxx \leftrightarrow Ryy) \vdash \exists y \forall x (Rxx \leftrightarrow Ryy)$  
(iv) $\vdash \forall x \exists y (Rxx \lor Ryy) \rightarrow \exists y \forall x (Rxx \lor Ryy)$
5. (a) Can the following English sentences be adequately formalized in $\mathcal{L}_=$ using the dictionary below? Provide the formalization or explain why not.

$P_1$: . . . is a Head of House

$P_2$: . . . is a Senior Tutor

$P_3$: . . . is a Fellow

$R_1$: . . . drinks port with . . .

$R_2$: . . . dines with . . .

(i) Some Heads of House drink port with some Senior Tutors. [2]

(ii) The Head of House dines with every Fellow. [3]

(iii) The Fellow who drinks port with every Senior Tutor is a Senior Tutor. [3]

(iv) The Senior Tutor who dines with the Fellow who drinks port with every Head of House only drinks port with Fellows. [4]

(b) Can the following English sentences be adequately formalized in $\mathcal{L}_=$ using the dictionary below? Provide the formalization or explain why not.

$P$: . . . is a panda

(i) There are at least two pandas. [2]

(ii) There is a prime number of pandas. [2]

(iii) The number of pandas is a prime number less than five. [4]

(c) “The language $\mathcal{L}_=$ offers no real advantage over the language $\mathcal{L}_2$ of predicate logic (without identity). We can always simply specify in the dictionary that a binary predicate $R$ formalizes ‘. . . is identical to . . .’.” Briefly discuss. [5]
SECTION B: GENERAL PHILOSOPHY

(Please use a separate booklet for each answer)

6. EITHER
   (a) How can you know that you are sitting an exam now if you don’t know that you are not dreaming?

   OR

   (b) ‘A clairvoyant lacks knowledge, even if clairvoyance is reliable. A five-year old child, however, can know that the door is shut or that the television is on.’ Discuss.

7. EITHER
   (a) What, if anything, is wrong with a circular justification of induction?

   OR

   (b) Does Hume’s appeal to custom constitute an adequate ‘sceptical solution’?

8. EITHER
   (a) What does Jackson’s Mary learn when she leaves her room?

   OR

   (b) Is a ‘real distinction’ between mind and body supported by what Descartes claims to be able to conceive?

9. EITHER
   (a) What, if anything, does Locke’s discussion of Day Man and Night Man tell us about personal identity?

   OR

   (b) Is your personal identity your animal identity?
10. EITHER

(a) Is Hume’s compatibilist view of freedom satisfactory?

OR

(b) Does distinguishing between causation and constraint allow for an adequate conception of freewill?

11. EITHER

(a) Is the free-will defence a satisfactory way of dealing with the problem of evil?

OR

(b) Compare and contrast Descartes’ two arguments for the existence of God in his *Meditations*.

SECTION C: MORAL PHILOSOPHY

*(Please use a separate booklet for each answer)*

12. Is Mill’s Hedonism consistent?

13. Are motives irrelevant to the morality of an action?

14. What is happiness? Is it a measurable quantity?

15. How important are our ordinary moral intuitions when evaluating a moral theory?

16. Is Utilitarianism too demanding? Or too permissive?

17. Should everyone’s pleasure and pain count equally towards the rightness or wrongness of an action?

[END OF PAPER]